1. Ideal Gas Law (Equation of state)

An *ideal gas* is an idealization in which the mean distance among molecules is far larger than the characteristic molecule size, and in which molecules collide elastically, that is without loss of energy or momentum. The atmospheric air is diluted enough to behave much like the ideal gas.

The ideal gas' pressure can be derived by assuming that we put a solid wall with known area A inside the gas and considering the force that bouncing molecules would exert on that wall. Applying the Newton's Law, we know that the force normal to the wall equals the rate of change of molecules' momentum projected on the normal direction to the wall (force and momentum are vectors). As direction of molecules is random, we can assume that at any time 1/3 of all molecules have the velocity component normal to the wall. Also, half of those molecules move away from the wall, so only 1/6 of all the molecules in the gas should be considered. Let's assume that molecules with mass m move with some average normal-to-the-wall *translational* velocity v. The collision with the wall would mean the change of molecules' momentum from mv to $-mv$, so the absolute momentum change will be $mv-(-mv)=2mv$. The number of such hits per *unit time* is equal to the total number of molecules in volume vA divided by 6, that is $\frac{1}{6}vAn$, where n is the given number of molecules per unit volume. Thus, the force acting on the wall is

$$F = 2mvvAn/6 = \frac{1}{3}mv^2An$$

Pressure is defined as force per unit area, so dividing by A we get

$$p = \frac{1}{3}nmv^2$$

Let's now define *temperature* T as the measure of mean kinetic energy of each molecule as

$$\frac{1}{2}mv^2 = \frac{3}{2}kT$$

where k is the Boltzmann constant (=1.38x10^{-23} m^2 kg s^{-2} K^{-1}). Then the above equation for the pressure of an ideal gas is written as

$$p = nkT$$

Concentration n is the number of molecules in unit volume. Such number of molecules equals the number of moles n_m per unit volume times Avagadro number N_a (=6.022x10^{23} mol^{-1}), so that

$$p = n_m N_a kT$$

or, introducing the universal gas constant $R^* = N_a k$ (=8.314 J mol^{-1}K^{-1})

$$p = n_m R^* T$$

By multiplying both sides by some arbitrary volume V, we get

$$pV = n_V R^* T \tag{1.1}$$

where n_V is the number of moles of gas in that volume.

The equation in form (1.1) is not commonly used in meteorology, as there is no practical use for some arbitrary volume of air in the Earth atmosphere; instead, it is more customary to use the volume of air per unit mass of air, so-called *specific* volume.

Let's multiply and then divide the right-hand-side (r.h.s) of (1.1) by the molar mass M (for air M = 0.029 *kg mol^{-1}*):

$$pV = nR^*T = nM\frac{R^*}{M}T = m\frac{R^*}{M}T$$

Here, m is the mass of volume V. Let's define the *specific volume* v as

$$v = V/m$$

and also, the closely related air density ρ (mass of unit of volume of air) as

$$\rho = 1/v = m/V$$

Then the ideal-gas equation for dry air can simply be written as

$$\boxed{p = \rho RT} \tag{1.2}$$

where R = 287 J kg^{-1} K^{-1} is the gas constant for dry air. The equation that relates the main (or state) properties of gas such as temperature and pressure is called the *equation of state*.