7. Moisture Related Variables

Here, some useful moisture related variables are introduced.

Mixing ratio: ***q***

Mixing ratio is defined as the ratio of specific mass or density of the water vapor to the density of *dry* air:

$$q \equiv \frac{\rho_v}{\rho_d} \qquad (7.1)$$

According to the Dalton's Law, the air pressure of the mixture of ideal gases is the sum of pressures of its components, so that the equation of state is valid for each of the components. In our case of moist air, $p = p_d + e$, where the subscript *d* refers to the dry air. Hence, we can rewrite (7.1) as

$$q = \frac{e}{R_v T}\frac{RT}{p_d} = \frac{e}{p_d}\frac{R}{R_v} = \varepsilon \frac{e}{p-e}$$

where $\varepsilon = R/R_v \approx 0.62$. For typical atmospheric conditions, the highest values of vapor pressure in troposphere is over the ocean in Tropics, that is about 50 hPa, which is much smaller than the pressure of a few hundred hPa in the lower and middle troposphere. In the upper troposphere, the air pressure can be also small; however, because it is usually very cold up there, the vapor pressure is also very small, that is still much smaller than the air pressure. Therefor, it is quite safe to assume that

$$q = \varepsilon \frac{e}{p} \qquad (7.2)$$

Typical value of *q* near the surface at 0^oC is $2x10^{-3}$ kg_{H2O}/kg_{air}, and at 30^oC is $20x10^{-3}$ kg_{H2O}/kg_{air}.

Specific humidity: ***r***

The specific humidity is defined as the ratio of vapor density to the density of *moist* air:

$$r \equiv \frac{\rho_v}{\rho} \qquad (7.3)$$

It is can be expressed in terms of the mixing ratio:

$$r = \frac{\rho_v}{\rho_v + \rho_d} = \frac{\rho_v}{\rho_d(1 + \frac{\rho_v}{\rho_d})} = \frac{q}{1+q} \approx q$$

and, because the mixing ratios in the atmosphere are typically much smaller than unity, the specific humidity is virtually undistinguishable from the mixing ratio.

Relative Humidity: ***RH***

The relative humidity is defined as the ratio of vapor pressure of the air to saturation vapor pressure at the air's temperature:

$$RH = \frac{e}{e_s} \tag{7.5}$$

By default, the saturation vapor pressure is assumed to be over liquid water. In the case of the saturation over ice, the corresponding quantity is called the *relative humidity over ice*:

$$RH_i = \frac{e}{e_{si}} \tag{7.6}$$

For given temperature, saturation vapor pressure over ice is always smaller than saturation vapor pressure over liquid water. As the result, the relative humidity over ice is always higher than 100% in so-called mixed-phase clouds. These clouds contain a mixture of ice crystals and cloud droplets, and typically exist in the range of temperatures from -40^{o}C to 0^{o}C. For the reasons that will be discussed in the corresponding chapter describing the cold-cloud microphysics, the vapor pressure in mixed clouds is always close to saturation over liquid water, that is, the relative humidity is 100%. Therefore, the relative humidity with respect to ice is higher than 100%; in fact, it can be as high as 150%! Under such conditions, the sublimation of vapor directly to ice is very efficient, so the ice content of mixed-phase clouds can rapidly increase at the expense of liquid water.

Liquid Water Content: q_l

The liquid water content is defined as the ratio of the density of liquid water in a cloud, that is mass of liquid water per unit volume of cloudy air, to the density of dry air:

$$q_l = \frac{\rho_l}{\rho_d} \approx \frac{\rho_l}{\rho} \tag{7.7}$$

Typical values of liquid water content in clouds are in the range 0.5-3 g/kg.

Ice Content: q_i

The ice content is defined as the ratio of the density of ice in a cloud, that is mass of ice crystals per unit volume of cloudy air, to the density of dry air:

$$q_i = \frac{\rho_i}{\rho_d} \approx \frac{\rho_i}{\rho} \tag{7.8}$$

Typical values of liquid water content in clouds are in the range 0.1-1 g/kg.

Total Water Content: q_t

The liquid water content is defined as the ratio of the density of total, vapor, liquid and solid, water in a cloud, to the density of dry air:

$$q_t = q + q_l + q_i \tag{7.9}$$

Obviously, unless some water precipitates our, the total water is conserved for all phase changes such as condensation, evaporation, sublimation, freezing, etc.

Virtual Temperature: T_v

The virtual temperature is the temperature that the dry air would have if its density and pressure were equal to the density of the moist air. To derive the expression for T_v, we use the Dalton's Law:

$$p = p_d + e = \rho_d RT + \rho_v R_v T = \rho_d RT(1 + \frac{\rho_v R_v}{\rho_d R}) = \rho RT \frac{\rho_d}{\rho_d + \rho_d}(1 + \frac{q}{\varepsilon}) = \rho RT \frac{1 + \frac{q}{\varepsilon}}{1 + q}$$

Thus, the equation of state of moist air is

$$p = \rho R T_v \tag{7.10}$$

where the virtual temperature T_v is defined as

$$T_v = T \frac{1 + \frac{q}{\varepsilon}}{1 + q} \tag{7.11}$$

From basic Calculus, using the facts that for $q << 1$, $\frac{1}{1+q} \approx 1-q$, and $q^2<<q$

$$\frac{1+\frac{q}{\varepsilon}}{1+q} \approx (1+\frac{q}{\varepsilon})(1-q) = 1+\frac{q}{\varepsilon}-q-\frac{q^2}{\varepsilon} \approx 1+0.61q$$

Thus, (7.11) simplifies to

$$T_v = T(1+0.61q) \qquad (7.12)$$

From (7.11) and (7.12), we see that for the given pressure and temperature, more moist air would have smaller density than dryer air. Less dense air would then have the tendency to rise. Thus for all other conditions being equal, the moist air would be more buoyant than the dry air.

Density Temperature: T_ρ

The density temperature is a generalization of the concept of virtual temperature to the cloudy air. The suspended cloud particles increase the apparent density of air. However, in the equation of state only the air density is used. The density temperature as defined as the temperature that the dry air would have if its density and pressure were equal to the density of the cloudy air, that is

$$p = \rho R T_\rho \qquad (7.13)$$

where the density temperature T_ρ is defined as

$$T_\rho = T(1+0.61q-q_l-q_i) \qquad (7.14)$$

Thus, for the given temperature, pressure, and vapor mixing ratio, the cloudy air would be more dense, or heavier, than the cloud free air, and, hence, less buoyant. In fact, large number of cloud particles suspended in the air can overcome positive buoyancy due to temperature and vapor excess over the environment and, thus, can cause the cloudy air to subside.