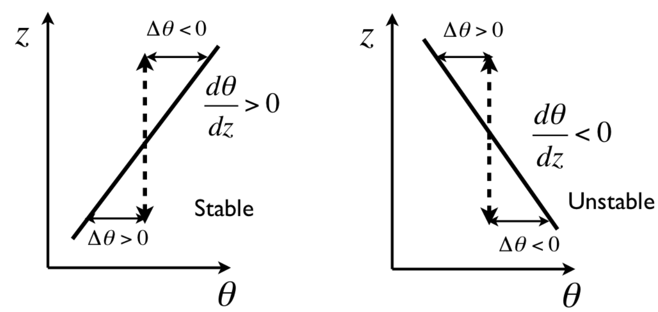
5. Static Stability of Dry Air

Consider a layer of air in hydrostatic balance satisfying (2.1). Let's assume that the temperature of air changes only with height z in such a way that the corresponding potential temperature θ increases with height, that is $\frac{d\theta}{dz} > 0$ as shown in the figure by the left diagram. Let's slowly displace a small parcel of air upward adiabatically. By slowly we mean that the pressure of the parcel has time to adjust to the environmental pressure, so that the pressure of the displaced parcel now is the same as pressure of the environment. As the displacement is done adiabatically, potential temperature of the parcel remains the same, that is, equal to the potential temperature the parcel had at the departure point. It is easy to see from the diagram that at its new position, the parcel would have lower potential temparture than the environment. As the pressures are the same, then, from definition of potential temperature (4.10), it follows that the kinetic (conventional) temperature of the parcel is now lower than that of the environment. From the ideal gas law, it then follows that the density of the parcel is higher than the density of the environment (as $\rho = p/(RT)$). This means that the parcel is no longer in hydrostatic balance as the vertical pressure gradient (left-hand-side of (2.1)) in the parcel is the same as that of the environment, but the density (right-hand-size of (2.1)) is now larger. Our parcel is now *relatively* heavier than the surrounding parcels of the environment, and hence, the parcel will have the tendency to sink, that is, to return to the departure point. If we now displace the parcel downward, then the parcel will be warmer then the environment, and, hence, not as dense, or lighter, and, therefore, will have the tendency to rise and return to the departure point.



Thus, we conclude that *for static stability, or stability of a layer of air in hydrostatic balance, with respect to small displacements in the vertical, the potential temperature in the layer should increase with height.*

If we now consider a situation when potential temperature of the layer decreases with height, that is $\frac{d\theta}{dz} < 0$, as shown on the figure above by the right diagram, then, using similar arguments, we can show that a parcel displaced upward will be warmer than the environment and, therefore, less dense, or lighter, than the environment. In other words, a parcel will have the tendency to rise even higher rather than return back to the departure point.

Thus, we conclude that *a layer in which the potential temperature decreases with increasing height is statically unstable with respect to small displacements in the vertical.*

A layer that is statically stable is called *stably stratified*, while a layer that is statically unstable is said to be *unstably stratified*. The border case is when potential temperature of the layer is constant with height, that is $\frac{d\theta}{dz} = 0$. Then, it is said that such a layer is *neutrally stratified*, or it has neutral stratification.

What happens to the unstably stratified layer? Well, it will start overturning or convecting, generating turbulence, gradually mixing the potential temperature in the vertical, until instability is removed, and the layer becomes close to being neutrally stratified, or *well-mixed* in terms of potential temperature. The viscosity will continuously remove kinetic energy and turn it into heat, so eventually the turbulence dies.

As was mentioned above, the border state between the statically stable and unstable stratification is the neutral stratification. What is the corresponding rate of change with height or *lapse rate* of the temperature? Let's derive the relationship between $\frac{d\theta}{dz}$ and $\frac{dT}{dz}$ in the layer in hydrostatic balance. For that, we would need to differentiate the definition of potential temperature (4.8) with respect to height *z*.

The easiest way to do so is to use the technique of differentiating-by-logarithms. Let's say that we have some functional dependence among variables *f*, *g* and *h* in the form:

$$f = g^a h^b \qquad (5.1)$$

where a and b are some constant coefficients. Let's now apply natural log to both sides of the above equation using the identities

$\ln(xy) = \ln(x) + \ln(y)$, and $\ln(x^a) = a\ln(x)$, so that, $\ln(f) = a\ln(g) + b\ln(h)$

Let's now differential with respect to *z*: $\frac{1}{f}\frac{df}{dz} = \frac{a}{g}\frac{dg}{dz} + \frac{b}{h}\frac{dh}{dz}$. Note that the variables that used to multiply are now added. We separated them! This is how one can differentiate logarithmically a function in the general form (5.1).

Let's now apply this technique to definition of the potential temperature (4.8). We have:

$$\frac{1}{\theta}\frac{d\theta}{dz} = \frac{1}{T}\frac{dT}{dz} - \frac{R}{c_p p}\frac{dp}{dz}$$

(obviously, the reference pressure p_0 has disappeared as it's a constant, and its derivative is zero). Using the hydrostatic equation (2.1) and the equation of state (1.2), we can derive

$$\frac{T}{\theta}\frac{d\theta}{dz} = \frac{dT}{dz} + \frac{g}{c_p} \qquad (5.2)$$

Try to derive (5.2) without the 'differentiating-by-logarithm' technique. It would be much harder! From (5.2), it follows that for neutrally stratified air

$$\frac{dT}{dz} = -\frac{g}{c_p}$$

With the value of g approximately equal to 10 m/s^2 and c_p about 1000 J/kg/K, the temperature lapse rate is about -10 K/km, that is the temperature in the neutrally stratified layer would decrease by about 10 K every kilometer of height increase. This lapse rate $\Gamma_a = -\frac{g}{c_p}$ is called *adiabatic lapse rate*. From (5.2), it also clear that for the local static stability of air, the local lapse rate should be larger than Γ_a, that is temperature should decrease with height slower than the adiabatic lapse rate. Note that if temperature increases with height, than the layer of air is always stable. For example, in troposphere, the temperature typically decreases with height with the lapse rate of about -7 K/km, which is larger than the adiabatic lapse rate. It makes the troposphere statically stable, so no *dry* convection (without clouds) generally occurs. The convection is only possible when clouds are formed due to a so-called *conditional instability of moist air*, discussed later. In stratosphere, the air temperature increases with height, which makes the stratosphere very stable. In hot summer afternoon, close to the surface, the lapse rate can exceed the adiabatic, and dry convection becomes possible in the lower troposphere, mixing the air in the vertical, and hence, removing the instability.

Note, the equation (5.2) can also be applied to compute the temperature of an air parcel, which is rising or descending adiabatically. In that case, the potential temperature is conserved, and hence, does not change with height, so that the temperature of the air parcel will change according to the dry adiabatic lapse rate. For example, the adiabatically rising parcel will cool by about 10 K per each 1 km of ascent. Conversely, subsiding air would warm-up by the same rate. The physical reason for the cooling is because the pressure is decreasing with height, so the air parcel expands and, hence, does work against the environment, which cools the parcel. Reversely, during the descent, the pressure increases, and now the environment does the work compressing the air parcel, which increases parcel's temperature.