13. Growth of Rain Drops by Collision-Coalescence

One of the outstanding problems of cloud physics is to explain the rapid development of rain showers from cumulus clouds with tops below freezing level. It has been observed that such clouds can produce rain showers in just under 20 min after their formation. It is clear that condensation itself is too slow to grow raindrops, which typically have radius of 1 mm = 1000 μm, which is 100 times larger than typical 10 μm radius of cloud droplets. The reason is that the rate of growth of droplets' size is inversely proportional to the size itself, that is, the bigger the droplet, the slower it grows. For example, given the typical equilibrium supersaturation of 0.1%, it would take 15 min for a droplet of 1 μm initial radius to grow to 10 μm radius, and 6 hours to grow to 50 μm radius, which is not even drizzle yet, that is, much smaller than the required raindrop size. Therefore, the only way for raindrops to appear in warm clouds is through collision of droplets among themselves. Note that not all collisions would end up with the participating droplets merging or *coalescing*. The process of rain production through collision and subsequent coalescence is called *collision-coalescence* process.

The cloud droplets are tiny; therefore, they have little inertia to overcome viscosity forces, and their trajectories generally closely follow the airflow streamlines. The flow streamlines never intersect, and so would do the droplets' trajectories; however, they are subject to gravitational settling. The bigger the droplet is, the faster its terminal velocity becomes. For tiny cloud droplets, the terminal velocity is given by the Stock's Law, which in the case of cloud droplets falling through the air gives the quadratic dependence of terminal velocity on droplet's radius:

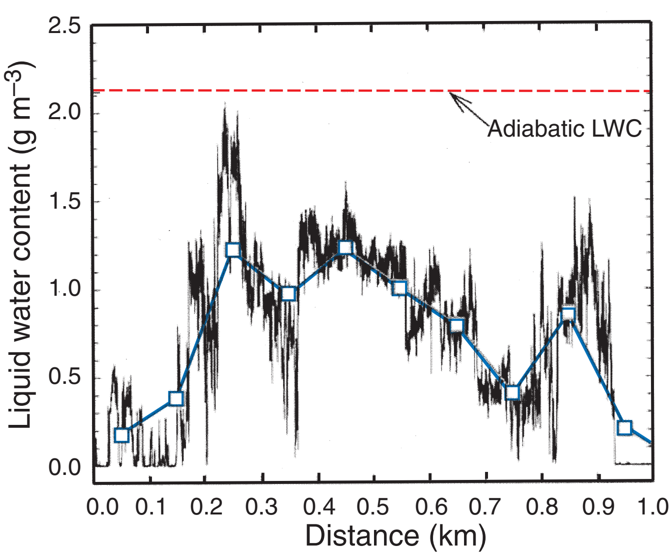
$$v_t = k_s r^2$$

where k_s = 1.19x10^6 cm^{-1} s^{-1}. Thus, the trajectories of droplets can intersect if these droplets have different sizes. In addition to tiny sizes, cloud droplets are very far away from each other. For the droplet concentration in a cloud is 100 cm^{-3}, which is rather typical for precipitating maritime cumulus clouds like those around Hawaii, the average distance between them would be 2000 μm. That is, for typical 10-μm-radius cloud droplet, the average droplet separation would be 100 time their size. Which makes it quite unlikely for them to collide due to just random jitter.

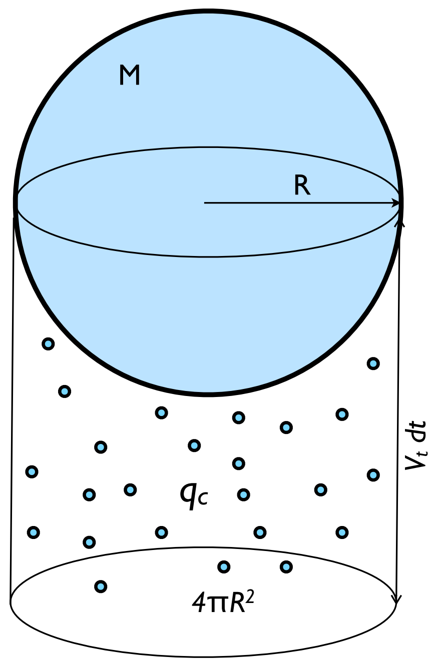
The only way to increase probability of collision is to make the difference in droplets' terminal velocity larger, that is, to make droplets have sizes as diverse as possible. Besides, increasing droplets' size also would generally make it more likely for them to coalesce. The collisions would make even larger droplets, and the collision process may become even more efficient and may even accelerate. The problem though is that the condensation alone does exactly the opposite, that is, it makes the cloud droplets be the same size, or, as it is said, condensation makes the size distribution narrow. There should be some processes that make the droplets differ in size, that is, to broaden the droplets' size distribution.

There are several ways to make relatively large droplets in clouds. For example, it has been proposed that some of the droplets could be activated on so-called giant CCN, for example, large sea salt particles. As the result, those droplets would be already several microns larger than the other droplets, which may jump-start the collision 'chain- reaction'. The typical concentration of raindrops in warm cloud is about 1 per liter compared to several million of 'normal' CCN, so it would take just a small fraction of CCN that are giant to produce rain. However, the problem with that theory is that observations suggest that generally there aren't enough of them in the air, although, in some instances, the giant CCN may indeed be the prime reason of early rain production. The lack of giant CCN is because of their quick removal from the air due to high gravitational settling rates.

Another process that can explain broad spectra needed to increase probability of collisions among cloud droplets is turbulent mixing. Clouds are always turbulent. The updraft generates hydrodynamic shear instabilities that produce turbulent eddies surrounding the main updraft. These eddies incorporate or *entrain* the environmental air into the cloud, diluting its water content. Generally, the environment above the boundary layer is very dry as it is much colder there than near the surface. As the result, one would expect that the cloud water in entraining cloud would be smaller than predicted by the adiabatic-parcel theory. It is supported by aircraft observations. For example, a figure below shows the liquid water content (LWC) along the path of an airplane penetrating through a cumulus cloud. One can see that most of the measured LWC is much smaller than the adiabatic LWC, especially near the edges. One can also see that there is a narrow region where LWC is close to adiabatic. That's indicates the position of the updraft *core*, which is mostly undiluted by the *entrainment* of the environmental air.



The result of entrainment is large diversity of water content and cloud drop sizes throughout the cloud. The turbulence further mixes the cloudy air parcels from different parts of the cloud with different histories. The result of turbulent mixing is large diversity of cloud drop sizes that came from different parts of cloud. This diversity can promote collision among droplets. It should be noted though that very small droplets, smaller than about 5 μm, have difficulty colliding or coalescing, as they are so tiny that generally follow the flow around other droplets, and, hence, are unlikely to collide. As the consequence, the droplets should grow larger than 5 μm to initiate or maintain the rain production; therefore, the clouds should have large enough LWC to produce rain or drizzle. This explains why small fair-weather cumulus clouds generally do not produce rain despite being turbulent.



Let us now assume that we have a raindrop of radius R falling with terminal velocity v_t through a field of cloud droplets with the cloud-water mixing ratio $q_c = \rho_c / \rho$, and which are much smaller than the falling raindrop. Then, we can ignore individual collisions and approximate this process as continuous growth of a drop's mass M by the amount dM after sweeping the droplets in a imaginary cylinder with the base equal to the drop's cross-section πR^2 and height $v_t dt$. The mass of collected cloud water is simply the volume of that cylinder times the density of cloud water ρ_c, that is

$$dM = \rho q_c \pi R^2 E_c v_t dt \tag{13.1}$$

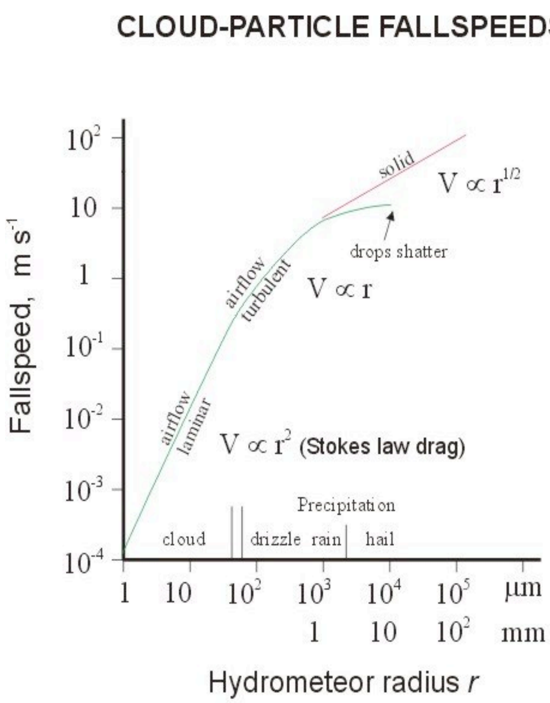
where E_c is *collection efficiency*, which is the measure of fraction of droplets that have collided and coalesced with the falling raindrop (some droplets can still go around and some may be deflected after collision). Generally, for large drops (larger than about 100 μm), the collection efficiency is about unity. Rewriting (13.1) as

$$\frac{dM}{dt} = 4\pi R^2 \rho_L \frac{dR}{dt} = \rho q_c \pi R^2 E_c v_t$$

we can get the following formula

$$\frac{dR}{dt} = \frac{\rho q_c E_c v_t}{4\rho_L} \tag{13.2}$$

The terminal velocity v_t of drops generally increases with their size as shown in figure below. Therefore, the rate of growth of a drops size *increases* with drops size, which is opposite to the condensational growth, that is, the growth of a raindrop by collision-coalescence accelerates with time rather than slows down!



Let us now rewrite (13.2) as the rate of change of drop's radius with height rather than time. For that, let's recall that the terminal velocity is just a rate of change of height with time: $v_t = \frac{dz}{dt}$. Substituting into (13.2) we immediately get

$$\frac{dR}{dz} = \frac{\rho q_c E_c}{4\rho_L} \tag{13.3}$$

Assuming that $E_c = 1$, and then integrating from some initial raindrop radius R_o to the radius R at the end of its fall through a cloud layer of thickness h, we get the change of raindrop's radius to be

$$R - R_o = \frac{1}{4\rho_L}\int_o^h \rho q_c \, dz \tag{13.4}$$

The quantity $\int_o^h \rho q_c \, dz$ is called *liquid water path* (LWP) and, by looking at the units, it is the total mass of the cloud water per unit area in a cloudy layer of thickness h. Thus, the change of raindrop's radius growing by continuous collection of cloud water in a cloud layer with given LWP depends only on that path, and does not depend on the raindrop's initial radius!

This is hardly intuitive. Let's assume that we have two raindrops, with radii of 1 and 2 mm, starting falling through a cloud layer with LWP that would make them grow by the same 1 mm. That means that 1-mm drop would increase its mass 2^3=8 times over initial, while the 2-mm one would increase it only by 1.5^3 = 3.4 times over initial. So, 1 ton of rainwater started as 1 mm drops would yield 7 tones of new rainwater, while the same 1 ton of rainwater but composed of 2-mm drops would yield only 2.4 tones of new rainwater, both falling through the same cloud layer!